4.28 For a BCC single crystal, would you expect the surface energy for a (100) plane to be greater or less than that for a (110) plane? Why? (Note: You may want to consult the solution to Problem 3.55 at the end of Chapter 3.)

Solution

The surface energy for a crystallographic plane will depend on its packing density [i.e., the planar density (Section 3.11)]—that is, the higher the packing density, the greater the number of nearest-neighbor atoms, and the more atomic bonds in that plane that are satisfied, and, consequently, the lower the surface energy. From the solution to Problem 3.55, the planar densities for BCC (100) and (110) are \( \frac{3}{16R^2} \) and \( \frac{3}{8R^2\sqrt[4]{2}} \), respectively—that is \( 0.19 \frac{R^2}{R^2} \) and \( 0.27 \frac{R^2}{R^2} \). Thus, since the planar density for (110) is greater, it will have the lower surface energy.

4.33 (a) Employing the intercept technique, determine the average grain size for the steel specimen whose microstructure is shown in Figure 9.25(a); use at least seven straight-line segments.

(b) Estimate the ASTM grain size number for this material.

Solution

(a) Below is shown the photomicrograph of Figure 9.25(a), on which seven straight line segments, each of which is 60 mm long has been constructed; these lines are labeled “1” through “7”.

In order to determine the average grain diameter, it is necessary to count the number of grains intersected by each of these line segments. These data are tabulated below.
<table>
<thead>
<tr>
<th>Line Number</th>
<th>No. Grains Intersected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

The average number of grain boundary intersections for these lines was 8.7. Therefore, the average line length intersected is just

\[
\frac{60 \text{ mm}}{8.7} = 6.9 \text{ mm}
\]

Hence, the average grain diameter, \( d \), is

\[
d = \frac{\text{ave. line length intersected}}{\text{magnification}} = \frac{6.9 \text{ mm}}{90} = 0.077 \text{ mm}
\]

(b) This portion of the problem calls for us to estimate the ASTM grain size number for this same material. The average grain size number, \( n \), is related to the number of grains per square inch, \( N \), at a magnification of 100× according to Equation 4.16. However, the magnification of this micrograph is not 100×, but rather 90×. Consequently, it is necessary to use Equation 4.17

\[
N_M \left(\frac{M}{100}\right)^2 = 2^{n-1}
\]

where \( N_M \) is the number of grains per square inch at magnification \( M \), and \( n \) is the ASTM grain size number. Taking logarithms of both sides of this equation leads to the following:

\[
\log N_M + 2 \log \left(\frac{M}{100}\right) = (n - 1) \log 2
\]

Solving this expression for \( n \) gives

\[
n = \frac{\log N_M + 2 \log \left(\frac{M}{100}\right)}{\log 2} + 1
\]
The photomicrograph on which has been constructed a square 1 in. on a side is shown below.

From Figure 9.25(a), $N_M$ is measured to be approximately 7, which leads to

$$n = \frac{\log 7 + 2 \log \left( \frac{90}{100} \right)}{\log 2} + 1$$

$$= 3.5$$

5.22 The diffusion coefficients for silver in copper are given at two temperatures:

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$D$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>$5.5 \times 10^{-16}$</td>
</tr>
<tr>
<td>900</td>
<td>$1.3 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

(a) Determine the values of $D_0$ and $Q_d$.
(b) What is the magnitude of $D$ at 875°C?

Solution

(a) Using Equation 5.9a, we set up two simultaneous equations with $Q_d$ and $D_0$ as unknowns as follows:
\[ \ln D_1 = \ln D_0 - \frac{Q_d}{R} \left( \frac{1}{T_1} \right) \]

\[ \ln D_2 = \ln D_0 - \frac{Q_d}{R} \left( \frac{1}{T_2} \right) \]

Solving for \( Q_d \) in terms of temperatures \( T_1 \) and \( T_2 \) (923 K [650°C] and 1173 K [900°C]) and \( D_1 \) and \( D_2 \) \((5.5 \times 10^{-16} \text{ and } 1.3 \times 10^{-13} \text{ m}^2/\text{s})\), we get

\[ Q_d = -R \frac{\ln D_1 - \ln D_2}{\frac{1}{T_1} - \frac{1}{T_2}} \]

\[ = -\frac{(8.31 \text{ J/mol-K}) \left[ \ln \left(5.5 \times 10^{-16}\right) - \ln \left(1.3 \times 10^{-13}\right)\right]}{\frac{1}{923 \text{ K}} - \frac{1}{1173 \text{ K}}} \]

\[ = 196,700 \text{ J/mol} \]

Now, solving for \( D_0 \) from Equation 5.8 (and using the 650°C value of \( D \))

\[ D_0 = D_1 \exp \left( \frac{Q_d}{RT_1} \right) \]

\[ = \left(5.5 \times 10^{-16} \text{ m}^2/\text{s}\right) \exp \left[ \frac{196,700 \text{ J/mol}}{(8.31 \text{ J/mol-K})(923 \text{ K})} \right] \]

\[ = 7.5 \times 10^{-5} \text{ m}^2/\text{s} \]

(b) Using these values of \( D_0 \) and \( Q_d \), \( D \) at 1148 K (875°C) is just

\[ D = \left(7.5 \times 10^{-5} \text{ m}^2/\text{s}\right) \exp \left[ -\frac{196,700 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1148 \text{ K})} \right] \]

\[ = 8.3 \times 10^{-14} \text{ m}^2/\text{s} \]
Note: this problem may also be solved using the “Diffusion” module in the VMSE software. Open the “Diffusion” module, click on the “D0 and Qd from Experimental Data” submodule, and then do the following:

1. In the left-hand window that appears, enter the two temperatures from the table in the book (converted from degrees Celsius to Kelvins) (viz. “923” (650°C) and “1173” (900°C), in the first two boxes under the column labeled “T (K)”. Next, enter the corresponding diffusion coefficient values (viz. “5.5e-16” and “1.3e-13”).

3. Next, at the bottom of this window, click the “Plot data” button.

4. A log D versus 1/T plot then appears, with a line for the temperature dependence for this diffusion system. At the top of this window are give values for $D_0$ and $Q_d$; for this specific problem these values are $7.55 \times 10^{-5} \text{ m}^2/\text{s}$ and 196 kJ/mol, respectively.

5. To solve the (b) part of the problem we utilize the diamond-shaped cursor that is located at the top of the line on this plot. Click-and-drag this cursor down the line to the point at which the entry under the “Temperature (T):” label reads “1148” (i.e., 875°C). The value of the diffusion coefficient at this temperature is given under the label “Diff Coeff (D):”. For our problem, this value is $8.9 \times 10^{-14} \text{ m}^2/\text{s}$. 