Fatigue strength fraction, $f$, of $S_{ut}$ at $10^3$ cycles for $S_e = S_a = 0.5 S_{ut}$.

The process given for finding $f$ can be repeated for various ultimate strengths. Figure 6–18 is a plot of $f$ for $70 \leq S_{ul} \leq 200$ kpsi. To be conservative, for $S_{ul} < 70$ kpsi, let $f = 0.9$.

For an actual mechanical component, $S_e$ is reduced to $S_e$ (see Sec. 6–9) which is less than $0.5 S_{ul}$. However, unless actual data is available, we recommend using the value of $f$ found from Fig. 6–18. Equation (a), for the actual mechanical component, can be written in the form

$$S_f = a N^b \tag{6-13}$$

where $N$ is cycles to failure and the constants $a$ and $b$ are defined by the points $10^3$, $(S_f)_{10^3}$ and $10^6$, $S_e$ with $(S_f)_{10^3} = f S_{ul}$. Substituting these two points in Eq. (6–13) gives

$$a = \frac{(f S_{ul})^2}{S_e} \tag{6-14}$$

$$b = -\frac{1}{3} \log \left( \frac{f S_{ul}}{S_e} \right) \tag{6-15}$$

If a completely reversed stress $\sigma_a$ is given, setting $S_f = \sigma_a$ in Eq. (6–13), the number of cycles-to-failure can be expressed as

$$N = \left( \frac{\sigma_a}{a} \right)^{1/b} \tag{6-16}$$

Low-cycle fatigue is often defined (see Fig. 6–10) as failure that occurs in a range of $1 \leq N \leq 10^3$ cycles. On a loglog plot such as Fig. 6–10 the failure locus in this range is nearly linear below $10^3$ cycles. A straight line between $10^3$, $f S_{ul}$ and 1, $S_{ul}$ (transformed) is conservative, and it is given by

$$S_f \geq S_{ul} N^{\log f/3} \quad 1 \leq N \leq 10^3 \tag{6-17}$$
EXAMPLE 6-2  Given a 1050 HR steel, estimate
(a) the rotating-beam endurance limit at $10^6$ cycles.
(b) the endurance strength of a polished rotating-beam specimen corresponding to $10^4$
   cycles to failure
(c) the expected life of a polished rotating-beam specimen under a completely reversed
   stress of 55 kpsi.

Solution
(a) From Table A-20, $S_{ut} = 90$ kpsi. From Eq. (6-8),
$$S' = 0.5(90) = 45 \text{ kpsi}$$

(b) From Fig. 6-18, for $S_{ut} = 90$ kpsi, $f = 0.86$. From Eq. (6-14),
$$a = \frac{[0.86(90)^2]}{45} = 133.1 \text{ kpsi}$$
From Eq. (6-15),
$$b = \frac{1}{3} \log \left[ \frac{0.86(90)}{45} \right] = -0.0785$$
Thus, Eq. (6-13) is
$$S'_f = 133.1 N^{-0.0785}$$

(c) From Eq. (6-16), with $\sigma_a = 55$ kpsi,
$$N = \left( \frac{55}{133.1} \right)^{1/-0.0785} = 77500 = 7.75(10^4) \text{cycles}$$

Answer
For $10^4$ cycles to failure, $S'_f = 133.1(10^4)^{-0.0785} = 64.6$ kpsi

Keep in mind that these are only estimates. So expressing the answers using three-place
accuracy is a little misleading.

6-9  Endurance Limit Modifying Factors
We have seen that the rotating-beam specimen used in the laboratory to determine
endurance limits is prepared very carefully and tested under closely controlled condi-
tions. It is unrealistic to expect the endurance limit of a mechanical or structural mem-
ber to match the values obtained in the laboratory. Some differences include
• Material: composition, basis of failure, variability
• Manufacturing: method, heat treatment, fretting corrosion, surface condition, stress
   concentration
• Environment: corrosion, temperature, stress state, relaxation times
• Design: size, shape, life, stress state, stress concentration, speed, fretting, galling
Marin\textsuperscript{12} identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. The question of whether to adjust the endurance limit by subtractive corrections or multiplicative corrections was resolved by an extensive statistical analysis of a 4340 (electric furnace, aircraft quality) steel, in which a correlation coefficient of 0.85 was found for the multiplicative form and 0.40 for the additive form. A Marin equation is therefore written as

\[ S_e = k_a k_b k_c k_d k_e S'_e \]  \hspace{1cm} (6-18)

where

- \( k_a \) = surface condition modification factor
- \( k_b \) = size modification factor
- \( k_c \) = load modification factor
- \( k_d \) = temperature modification factor
- \( k_e \) = reliability factor\textsuperscript{13}
- \( k_f \) = miscellaneous-effects modification factor
- \( S'_e \) = rotary-beam test specimen endurance limit
- \( S_e \) = endurance limit at the critical location of a machine part in the geometry and condition of use

When endurance tests of parts are not available, estimations are made by applying Marin factors to the endurance limit.

\textbf{Surface Factor \( k_a \)}

The surface of a rotating-beam specimen is highly polished, with a final polishing in the axial direction to smooth out any circumferential scratches. The surface modification factor depends on the quality of the finish of the actual part surface and on the tensile strength of the part material. To find quantitative expressions for common finishes of machine parts (ground, machined, or cold-drawn, hot-rolled, and as-forged), the coordinates of data points were recaptured from a plot of endurance limit versus ultimate tensile strength of data gathered by Lipson and Noll and reproduced by Horger.\textsuperscript{14} The data can be represented by

\[ k_a = a S_{ut}^b \]  \hspace{1cm} (6-19)

where \( S_{ut} \) is the minimum tensile strength and \( a \) and \( b \) are to be found in Table 6–2.


\textsuperscript{13}Complete stochastic analysis is presented in Sec. 6–17. Until that point the presentation here is one of a deterministic nature. However, we must take care of the known scatter in the fatigue data. This means that we will not carry out a true reliability analysis at this time but will attempt to answer the question: What is the probability that a \textit{known} (assumed) stress will exceed the strength of a randomly selected component made from this material population?

Table 6-2
Parameters for Marin Surface Modification Factor, Eq. (6-19)

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>Factor a</th>
<th>$S_{ut}$, kpsi</th>
<th>$S_{ut}$, MPa</th>
<th>Exponent b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>1.34</td>
<td>1.58</td>
<td>-0.085</td>
<td></td>
</tr>
<tr>
<td>Machined or cold-drawn</td>
<td>2.70</td>
<td>4.51</td>
<td>-0.265</td>
<td></td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>14.4</td>
<td>57.7</td>
<td>-0.718</td>
<td></td>
</tr>
<tr>
<td>As-forged</td>
<td>39.9</td>
<td>272.</td>
<td>-0.995</td>
<td></td>
</tr>
</tbody>
</table>


EXAMPLE 6-3
A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate $k_a$.

Solution
From Table 6-2, $a = 4.51$ and $b = -0.265$. Then, from Eq. (6-19)

$$k_a = 4.51(520)^{-0.265} = 0.860$$

Answer

Again, it is important to note that this is an approximation as the data is typically quite scattered. Furthermore, this is not a correction to take lightly. For example, if in the previous example the steel was forged, the correction factor would be 0.540, a significant reduction of strength.

Size Factor $k_b$
The size factor has been evaluated using 133 sets of data points. The results for bending and torsion may be expressed as

$$k_b = \begin{cases} 
(d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\
0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\
(d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\
1.51d^{-0.157} & 51 < d \leq 254 \text{ mm}
\end{cases}$$

(6-20)

For axial loading there is no size effect, so

$$k_b = 1$$

(6-21)

but see $k_c$.

One of the problems that arises in using Eq. (6-20) is what to do when a round bar in bending is not rotating, or when a noncircular cross section is used. For example, what is the size factor for a bar 6 mm thick and 40 mm wide? The approach to be used

---

here employs an effective dimension $d_e$ obtained by equating the volume of material stressed at and above 95 percent of the maximum stress to the same volume in the rotating-beam specimen.\textsuperscript{16} It turns out that when these two volumes are equated, the lengths cancel, and so we need only consider the areas. For a rotating round section, the 95 percent stress area is the area in a ring having an outside diameter $d$ and an inside diameter of 0.95$d$. So, designating the 95 percent stress area $A_{0.95\sigma}$, we have

$$A_{0.95\sigma} = \frac{\pi}{4} [d^2 - (0.95d)^2] = 0.0766d^2$$  \hspace{1cm} \text{(6-22)}$$

This equation is also valid for a rotating hollow round. For nonrotating solid or hollow rounds, the 95 percent stress area is twice the area outside of two parallel chords having a spacing of 0.95$d$, where $d$ is the diameter. Using an exact computation, this is

$$A_{0.95\sigma} = 0.01046d^2$$ \hspace{1cm} \text{(6-23)}$$

with $d_e$ in Eq. (6-22), setting Eqs. (6-22) and (6-23) equal to each other enables us to solve for the effective diameter. This gives

$$d_e = 0.370d$$ \hspace{1cm} \text{(6-24)}$$

as the effective size of a round corresponding to a nonrotating solid or hollow round.

A rectangular section of dimensions $h \times b$ has $A_{0.95\sigma} = 0.05hb$. Using the same approach as before,

$$d_e = 0.808(hb)^{1/2}$$ \hspace{1cm} \text{(6-25)}$$

Table 6-3 provides $A_{0.95\sigma}$ areas of common structural shapes undergoing nonrotating bending.

---

### Table 6-3

**A\(_{0.95\sigma}\) Areas of Common Nonrotating Structural Shapes**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula 1</th>
<th>Formula 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round</td>
<td>(A_{0.95\sigma} = 0.01046d^2)</td>
<td>(d_e = 0.370d)</td>
</tr>
<tr>
<td>Square</td>
<td>(A_{0.95\sigma} = 0.05hb)</td>
<td>(d_e = 0.808\sqrt{hb})</td>
</tr>
<tr>
<td>T-shaped</td>
<td>(A_{0.95\sigma} = \begin{cases} 0.10a_t &amp; \text{axis 1-1} \ 0.05ba &amp; t_f &gt; 0.025a \end{cases})</td>
<td>(A_{0.95\sigma} = \begin{cases} 0.05ab &amp; \text{axis 1-1} \ 0.052x_s + 0.1t_f(b - x) &amp; \text{axis 2-2} \end{cases})</td>
</tr>
</tbody>
</table>

### Loading Factor \(k_c\)

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with \(S_m\). This is discussed further in Sec. 6-17. Here, we will specify average values of the load factor as

\[
k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}
\]

### Temperature Factor \(k_d\)

When operating temperatures are below room temperature, brittle fracture is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, yielding should be investigated first because the yield strength drops off so rapidly with temperature; see Fig. 2-9. Any stress will induce creep in a material operating at high temperatures; so this factor must be considered too.

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17Use this only for pure torsional fatigue loading. When torsion is combined with other stresses, such as bending, \(k_c = 1\) and the combined loading is managed by using the effective von Mises stress as in Sec. 5-5. Note: For pure torsion, the distortion energy predicts that \((k_c)_{\text{torsion}} = 0.577\).
Table 6-4

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>$S_T/S_{RT}$</th>
<th>Temperature, °F</th>
<th>$S_T/S_{RT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.000</td>
<td>70</td>
<td>1.000</td>
</tr>
<tr>
<td>50</td>
<td>1.010</td>
<td>100</td>
<td>1.008</td>
</tr>
<tr>
<td>100</td>
<td>1.020</td>
<td>200</td>
<td>1.020</td>
</tr>
<tr>
<td>150</td>
<td>1.025</td>
<td>300</td>
<td>1.024</td>
</tr>
<tr>
<td>200</td>
<td>1.020</td>
<td>400</td>
<td>1.018</td>
</tr>
<tr>
<td>250</td>
<td>1.000</td>
<td>500</td>
<td>0.995</td>
</tr>
<tr>
<td>300</td>
<td>0.975</td>
<td>600</td>
<td>0.963</td>
</tr>
<tr>
<td>350</td>
<td>0.943</td>
<td>700</td>
<td>0.927</td>
</tr>
<tr>
<td>400</td>
<td>0.900</td>
<td>800</td>
<td>0.872</td>
</tr>
<tr>
<td>450</td>
<td>0.843</td>
<td>900</td>
<td>0.797</td>
</tr>
<tr>
<td>500</td>
<td>0.768</td>
<td>1000</td>
<td>0.698</td>
</tr>
<tr>
<td>550</td>
<td>0.672</td>
<td>1100</td>
<td>0.567</td>
</tr>
<tr>
<td>600</td>
<td>0.549</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Data source: Fig. 2-9.

Finally, it may be true that there is no fatigue limit for materials operating at high temperatures. Because of the reduced fatigue resistance, the failure process is, to some extent, dependent on time.

The limited amount of data available show that the endurance limit for steels increases slightly as the temperature rises and then begins to fall off in the 400 to 700°F range, not unlike the behavior of the tensile strength shown in Fig. 2-9. For this reason it is probably true that the endurance limit is related to tensile strength at elevated temperatures in the same manner as at room temperature.\(^\text{18}\) It seems quite logical, therefore, to employ the same relations to predict endurance limit at elevated temperatures as are used at room temperature, at least until more comprehensive data become available. At the very least, this practice will provide a useful standard against which the performance of various materials can be compared.

Table 6-4 has been obtained from Fig. 2-9 by using only the tensile-strength data. Note that the table represents 145 tests of 21 different carbon and alloy steels. A fourth-order polynomial curve fit to the data underlying Fig. 2-9 gives

\[
k_d = 0.975 + 0.432 \times 10^{-3} T_F - 0.115 \times 10^{-5} T_F^2 + 0.104 \times 10^{-8} T_F^3 - 0.595 \times 10^{-12} T_F^4
\]  

(6-27)

where 70 ≤ $T_F$ ≤ 1000°F.

Two types of problems arise when temperature is a consideration. If the rotating-beam endurance limit is known at room temperature, then use

\[
k_d = \frac{S_T}{S_{RT}}
\]  

(6-28)

from Table 6–4 or Eq. (6–27) and proceed as usual. If the rotating-beam endurance limit is not given, then compute it using Eq. (6–8) and the temperature-corrected tensile strength obtained by using the factor from Table 6–4. Then use $k_d = 1$.

**EXAMPLE 6–5**

A 1035 steel has a tensile strength of 70 kpsi and is to be used for a part that sees 450°F in service. Estimate the Marin temperature modification factor and $(Se)_{450}$ if

(a) The room-temperature endurance limit by test is $(Se)_{70} = 39.0$ kpsi.

(b) Only the tensile strength at room temperature is known.

**Solution**

(a) First, from Eq. (6–27),

$$k_d = 0.975 + 0.432(10^{-3})(450) - 0.115(10^{-5})(450^2) + 0.104(10^{-8})(450^3) - 0.595(10^{-12})(450^4) = 1.007$$

Thus,

$$(Se)_{450} = kd(Se)_{70} = 1.007(39.0) = 39.3 \text{ kpsi}$$

(b) Interpolating from Table 6–4 gives

$$(ST/S_{RT})_{450} = 1.018 + (0.995 - 1.018) \frac{450 - 400}{500 - 400} = 1.007$$

Thus, the tensile strength at 450°F is estimated as

$$(S_{ul})_{450} = (ST/S_{RT})_{450} (S_{ul})_{70} = 1.007(70) = 70.5 \text{ kpsi}$$

From Eq. (6–8) then,

$$(Se)_{450} = 0.5 (S_{ul})_{450} = 0.5(70.5) = 35.2 \text{ kpsi}$$

**Answer**

Part a gives the better estimate due to actual testing of the particular material.

**Reliability Factor $k_e$**

The discussion presented here accounts for the scatter of data such as shown in Fig. 6–17 where the mean endurance limit is shown to be $S_e/S_{ul} = 0.5$, or as given by Eq. (6–8). Most endurance strength data are reported as mean values. Data presented by Haugen and Wirching$^{19}$ show standard deviations of endurance strengths of less than 8 percent. Thus the reliability modification factor to account for this can be written as

$$k_e = 1 - 0.08 z_a \quad (6–29)$$

where $z_a$ is defined by Eq. (20–16) and values for any desired reliability can be determined from Table A–10. Table 6–5 gives reliability factors for some standard specified reliabilities.

For a more comprehensive approach to reliability, see Sec. 6–17.

### Table 6-5

<table>
<thead>
<tr>
<th>Reliability %</th>
<th>Transformation Variate $z_\alpha$</th>
<th>Reliability Factor $k_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>90</td>
<td>1.288</td>
<td>0.897</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
<td>0.868</td>
</tr>
<tr>
<td>99</td>
<td>2.326</td>
<td>0.814</td>
</tr>
<tr>
<td>99.9</td>
<td>3.091</td>
<td>0.753</td>
</tr>
<tr>
<td>99.99</td>
<td>3.719</td>
<td>0.702</td>
</tr>
<tr>
<td>99.999</td>
<td>4.265</td>
<td>0.659</td>
</tr>
<tr>
<td>99.9999</td>
<td>4.753</td>
<td>0.620</td>
</tr>
</tbody>
</table>

### Figure 6-19

The failure of a case-hardened part in bending or torsion. In this example, failure occurs in the core.

### Miscellaneous-Effects Factor $k_f$

Though the factor $k_f$ is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for, because actual values of $k_f$ are not always available.

*Residual stresses* may either improve the endurance limit or affect it adversely. Generally, if the residual stress in the surface of the part is compression, the endurance limit is improved. Fatigue failures appear to be tensile failures, or at least to be caused by tensile stress, and so anything that reduces tensile stress will also reduce the possibility of a fatigue failure. Operations such as shot peening, hammering, and cold rolling build compressive stresses into the surface of the part and improve the endurance limit significantly. Of course, the material must not be worked to exhaustion.

The endurance limits of parts that are made from rolled or drawn sheets or bars, as well as parts that are forged, may be affected by the so-called *directional characteristics* of the operation. Rolled or drawn parts, for example, have an endurance limit in the transverse direction that may be 10 to 20 percent less than the endurance limit in the longitudinal direction.

Parts that are case-hardened may fail at the surface or at the maximum core radius, depending upon the stress gradient. Figure 6-19 shows the typical triangular stress distribution of a bar under bending or torsion. Also plotted as a heavy line in this figure are the endurance limits $S_e$ for the case and core. For this example the endurance limit of the core rules the design because the figure shows that the stress $\sigma$ or $\tau$, whichever applies, at the outer core radius, is appreciably larger than the core endurance limit.
Of course, if stress concentration is also present, the stress gradient is much steeper, and hence failure in the core is unlikely.

**Corrosion**

It is to be expected that parts that operate in a corrosive atmosphere will have a lowered fatigue resistance. This is, of course, true, and it is due to the roughening or pitting of the surface by the corrosive material. But the problem is not so simple as the one of finding the endurance limit of a specimen that has been corroded. The reason for this is that the corrosion and the stressing occur at the same time. Basically, this means that in time any part will fail when subjected to repeated stressing in a corrosive atmosphere. There is no fatigue limit. Thus the designer’s problem is to attempt to minimize the factors that affect the fatigue life; these are:

- Mean or static stress
- Alternating stress
- Electrolyte concentration
- Dissolved oxygen in electrolyte
- Material properties and composition
- Temperature
- Cyclic frequency
- Fluid flow rate around specimen
- Local crevices

**Electrolytic Plating**

Metallic coatings, such as chromium plating, nickel plating, or cadmium plating, reduce the endurance limit by as much as 50 percent. In some cases the reduction by coatings has been so severe that it has been necessary to eliminate the plating process. Zinc plating does not affect the fatigue strength. Anodic oxidation of light alloys reduces bending endurance limits by as much as 39 percent but has no effect on the torsional endurance limit.

**Metal Spraying**

Metal spraying results in surface imperfections that can initiate cracks. Limited tests show reductions of 14 percent in the fatigue strength.

**Cyclic Frequency**

If, for any reason, the fatigue process becomes time-dependent, then it also becomes frequency-dependent. Under normal conditions, fatigue failure is independent of frequency. But when corrosion or high temperatures, or both, are encountered, the cyclic rate becomes important. The slower the frequency and the higher the temperature, the higher the crack propagation rate and the shorter the life at a given stress level.

**Frettage Corrosion**

The phenomenon of frettage corrosion is the result of microscopic motions of tightly fitting parts or structures. Bolted joints, bearing-race fits, wheel hubs, and any set of tightly fitted parts are examples. The process involves surface discoloration, pitting, and eventual fatigue. The frettage factor $k_f$ depends upon the material of the mating pairs and ranges from 0.24 to 0.90.
Stress Concentration and Notch Sensitivity

In Sec. 3–13 it was pointed out that the existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increases the theoretical stresses significantly in the immediate vicinity of the discontinuity. Equation (3–48) defined a stress concentration factor $K_t$ (or $K_{ts}$), which is used with the nominal stress to obtain the maximum resulting stress due to the irregularity or defect. It turns out that some materials are not fully sensitive to the presence of notches and hence, for these, a reduced value of $K_t$ can be used. For these materials, the maximum stress is, in fact,

$$\sigma_{\text{max}} = K_f \sigma_0 \quad \text{or} \quad \tau_{\text{max}} = K_{fs} \tau_0$$  \hspace{1cm} (6-30)

where $K_f$ is a reduced value of $K_t$ and $\sigma_0$ is the nominal stress. The factor $K_f$ is commonly called a fatigue stress-concentration factor, and hence the subscript $f$. So it is convenient to think of $K_f$ as a stress-concentration factor reduced from $K_t$ because of lessened sensitivity to notches. The resulting factor is defined by the equation

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$  \hspace{1cm} (a)

Notch sensitivity $q$ is defined by the equation

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$  \hspace{1cm} (6-31)

where $q$ is usually between zero and unity. Equation (6–31) shows that if $q = 0$, then $K_f = 1$, and the material has no sensitivity to notches at all. On the other hand, if $q = 1$, then $K_f = K_t$, and the material has full notch sensitivity. In analysis or design work, find $K_t$ first, from the geometry of the part. Then specify the material, find $q$, and solve for $K_f$ from the equation

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$  \hspace{1cm} (6-32)

For steels and 2024 aluminum alloys, use Fig. 6-20 to find $q$ for bending and axial loading. For shear loading, use Fig. 6-21. In using these charts it is well to know that the actual test results from which the curves were derived exhibit a large amount of

---

**Figure 6-20**

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of $q$ corresponding to the $r = 0.1$ in (4 mm) ordinate. (From George Sines and J. L. Waisman (eds.), Metal Fatigue, McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)
scatter. Because of this scatter it is always safe to use $K_f = K_i$ if there is any doubt about the true value of $q$. Also, note that $q$ is not far from unity for large notch radii.

The notch sensitivity of the cast irons is very low, varying from 0 to about 0.20, depending upon the tensile strength. To be on the conservative side, it is recommended that the value $q = 0.20$ be used for all grades of cast iron.

Figure 6–20 has as its basis the Neuber equation, which is given by

$$K_f = 1 + \frac{K_i - 1}{1 + \sqrt{\alpha}/r} \tag{6-33}$$

where $\sqrt{\alpha}$ is defined as the Neuber constant and is a material constant. Equating Eqs. (6–31) and (6–33) yields the notch sensitivity equation

$$q = \frac{1}{1 + \sqrt{\alpha}/r} \tag{6-34}$$

For steel, with $S_{ut}$ in kpsi, the Neuber constant can be approximated by a third-order polynomial fit of data as

$$\sqrt{\alpha} = 0.245799 - 0.307794(10^{-2})S_{ut} + 0.150874(10^{-4})S_{ut}^2 - 0.266978(10^{-7})S_{ut}^3 \tag{6-35}$$

To use Eq. (6–33) or (6–34) for torsion for low-alloy steels, increase the ultimate strength by 20 kpsi in Eq. (6–35) and apply this value of $\sqrt{\alpha}$.

**EXAMPLE 6–6**

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate $K_f$ using:

(a) Figure 6–20.

(b) Equations (6–33) and (6–35).