SKETCHING: For all problems in engineering, be sure to make sketches as appropriate. Free body diagrams are almost always appropriate for stress analysis problems. For bending problems, shear and moment diagrams are appropriate.

GENERAL HINT: when doing stress analysis using metric units, most engineering materials have strength on the order of MPa (10^6 N/m^2). If you use “newtons” and “millimeters” as your units for force and length, this works out well as 1 N/mm^2 is 10^6 N/m^2 (1 N/mm^2 = 1MPa). Life is easier if you use N/mm^2.

SHARED WISDOM: always include units along each and every step of your calculations, and always show the equations you use in variable form before using numbers. This practice requires a small investment of time, but pays HUGE DIVIDENDS later! It will help you trouble shoot problems (find where you make mistakes) and help you 6 months from now remember what you were doing (critically important for practicing engineers). Now is the time to develop good habits.

1. Educational Purpose: This problem is meant to help you understand how to apply static failure theories which requires applying engineering judgment to make a decision.
   How much torque can a 0.500 inch diameter bar carry before failure for the following materials? Failure is defined as either yielding or fracture – which ever comes first. How will these two materials fail (yield or fracture)? What failure theory(s) would be appropriate to apply to each of these? For material properties see appendix A-24 or MatWeb (I’m okay with MatWeb material properties for SCHOOL WORK – NEVER for REAL ENGINEERING WORK!). Hint: if you find tensile strength but not yield strength, that should tell you something (there is probably a reason for lacking yield strength data).
   a) ASTM class 20 gray cast iron
   b) 2024-T3

The following two problems are from the textbook. There is a copy of them (PDF) on the course web page. Educational Purpose: these problems are meant to stretch you beyond classroom lectures. You will need to look up appropriate equations to solve the problems. These problems also introduce you to a design consideration (angular deflection) for bearings. We will discuss bearings only very briefly later this semester.

2. Problem 3-69
3. 4-36 (4-36 is based on 3-69). This problem may appear to be quite challenging; however, by “breaking” it into small pieces, each small piece should be manageable. You will need to use appropriate beam deflection charts that show angular deflection for various loads (pdf of such a chart is on the course web page in the homework table). NOTE: angles shown in such charts are typically in radians, not degrees. Here is what is required for this problem:
a) Use Excel (or MatLab, or similar) to create a graph which shows the relationship between angular deflection and shaft diameter at each of the two bearing locations for the following loading condition (from problem 4-36):

![Diagram showing loading condition with T1+T2, 230mm to 280mm, and symbol for bearings.]

b) Use Excel (or MatLab, or similar) to create a graph which shows the relationship between angular deflection and shaft diameter at each of the two bearing locations for the loading condition shown below (also from 4-36). Hint, it is unlikely you will find this loading condition in any beam deflection chart. You will have to think back (way back) to your days in statics class. Remember “equivalent systems”? Replace the cantilever load (1800N+270N) with an equivalent force and couple-moment…but where? Think about this…if you have the equivalent force and moment at a location such that the force will not cause any shaft deflection (hmmm, where?) then you won’t need to include that force in your deflection analysis; only the moment. Are there any loading conditions shown in the beam deflection chart that has only an applied moment…

![Diagram showing loading condition with 2070N, 510mm to 300mm.]

c) Hopefully, you have assumed that the shaft will remain linear elastic (not plastically deformed). If it is linear, then you can add parts (a) and (b) together to solve problem 4-36. Do so to answer 4-36, and show a single graph with three lines (one each from parts a, b, and c to this problem).

4. Educational purpose: help understand what happens to bending stress if the material is not linear-elastic. It is also a “precursor” to stress concentrations – helping you to identify they exist and are difficult to properly model with FEA.

For this problem, assume the aluminum alloy is elastic/perfectly plastic, and has yield strength of 300MPa (as shown in the $\sigma-\varepsilon$ diagram sketch below). Perfectly plastic means that once the yield stress is reached the stress remains constant beyond that. If you have not taken ME304, please state so in your answer. If that is the case, you do not need to create the FEA model, but you do need to get a copy of one from a classmate and have them explain it to you. You need to provide that copy as part of your answer (giving your classmate credit) and discuss the results as requested.
Using finite element analysis, determine the bending stress in the cantilever beam shown below. The load, P, is 5kN. Can this effectively be modeled in 2-dimensions only? (I think so).

a. Provide a hardcopy of the FEA results and briefly describe how the bending stress varies in the beam (from end to end, from top to bottom).

b. Where specifically do you expect yielding to occur first as the load is increased sufficiently? Briefly explain why.

c. For a section of the beam near the attachment (90mm from the loaded end), sketch the stress distribution from top to bottom through the thickness (hint: like Figure 3-14 in your text).

d. **For extra credit**: Repeat part “c”, but for a load of 10kN. Hint, the 5kN load will not cause yielding, but the 10kN load will cause yielding. An assumption that is typically made for beam bending: “the material is linear elastic”. Once yielding occurs, the assumption is not valid and therefore the equation: \( \sigma = \frac{My}{I} \) is no longer valid. Briefly explain the difference in the stress distribution between parts c and d. **You do NOT need to model this with FEA.**

Educational purpose: *learn the basics of power transmission through chain-drives, belt-drives, and gear trains. All three of those forms of power transmission share commonality in determining speed and torque relations. We will discuss gears to a limited extent later in the semester, but you will need to understand speed and torque relations sooner than that for the project.*

5. A sprocket (or sprocket-wheel) is the gear-like machine element used with chain-drives. Consider the two different chain-driven power trains (parts a and b below).
For both of these, the load transmitting force in the belt is 100 pounds (tension – you can’t push a rope…or a chain) and assume the “slack” side has no tension. Draw a free body diagram of both the driving and driven sprockets. Then determine the torque in the driving sprocket (the sprocket receiving power from the motor) and the torque in the driven sprocket (the power is transmitted to the driven sprocket from the driving sprocket through the chain).

a) The driving sprocket is 10” in diameter and the driven sprocket is 5” in diameter.

b) The driving sprocket is 5” in diameter and the driven sprocket is 10” in diameter.

6. Using a bicycle (yours or a friends), select the lowest gear-speed (what you would use for going up a steep hill). Answer the following
   a) How many “teeth” are on the driving sprocket, N_{in}? The “driving sprocket” is the sprocket attached to the pedals.
   b) What is the diameter of the driving sprocket, d_{in}? To determine the diameter of the sprocket, measure the distance from the center of the chain on one side of the sprocket to the center of the chain on the opposite side.
   c) What is the diametral pitch of the driving sprocket, P_{in}? The diametral pitch is the number of teeth per inch, P=N/d. N is the number teeth, d is the diameter of the sprocket.
   d) How many “teeth” are on the driven sprocket, N_{out}? The “driven sprocket” is the sprocket attached to the wheel.
   e) What is the diameter of the driven sprocket, d_{out}?
   f) What is the diametral pitch of the driven sprocket, P_{out}?
   g) If the pedals were being operated at 100RPM (revolutions per minute), what would the speed of the wheel be (in RPM)? Hint, if the tire rotates one-half revolution for each full pedal revolution, then the answer would be 50RPM.
   h) Based on your bike measurements above, show that the following is true:
      \[ P_{out} = P_{in} = P = \frac{N_{in}}{d_{in}} = \frac{N_{out}}{d_{out}} \]
   i) Show that the following is also true:
      \[ \frac{d_{in}}{d_{out}} = \frac{\omega_{out}}{\omega_{in}} \] where \( \omega_{in} \) and \( \omega_{out} \) are the angular velocities (RPM or radians per second) for the driving and driven sprockets, respectively.
   j) Assuming power loss in the chain-drive system is negligible (typically, a reasonable assumption), and power is torque times angular velocity (power = T\omega); therefore, T_{in}\omega_{in} = T_{out}\omega_{out}. If the bicyclist applies 50ft-lb torque to the pedal, what is the torque in the wheel?

7. Educational purpose: this problem builds on the previous problem. This problem now introduces power transmission. This too, will be important for the project.
   Gear ratios are expressed as fractions or ratios: \( \frac{\omega_{driving}}{\omega_{driven}} \) or typically \( \omega_{driving}:\omega_{driven} \) (input speed to output speed). A ratio of 2:1 means the output of the chain (or gear) tire is spinning at half the speed of the input sprocket (or gear). A ratio of 0.9:1 means the output (tire) is spinning faster than the input (feet-peddle) – this is sometimes referred to as “overdrive”. Consider an electric motor with the performance curve shown below. This is a typical curve for electric DC motors – i.e. there is a linear relation between speed (rpm) and torque (T). At a certain torque level, the motor stalls (stops turning) and spins at a maximum speed when no load is
applied (free spinning). Power is the product, $T\omega$. You are part of the design team for winch system.

Criterial Table

<table>
<thead>
<tr>
<th>#</th>
<th>Criteria</th>
<th>Priority</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Motor</td>
<td>Essential</td>
<td>Use motor performance curve provided</td>
</tr>
<tr>
<td>2</td>
<td>Drum</td>
<td>Essential</td>
<td>Diameter of the lifting drum is 20mm</td>
</tr>
<tr>
<td>3</td>
<td>Load</td>
<td>Essential</td>
<td>Capable of lifting a 1kg load</td>
</tr>
<tr>
<td>4</td>
<td>Speed</td>
<td>Important</td>
<td>Lifting the 1 kg load as fast as possible within the given constraints.</td>
</tr>
</tbody>
</table>

Your task is to determine the following based on the criteria in the Criterial Table:

a) What gear ratio best meets the needs?

b) How long will it take to raise the load 1 meter assuming constant velocity? Assume $a_g=10\text{m/s}^2$.

Electric motor performance curve.

8. Educational purpose: *mechanical engineering has its own jargon. It is important for students to be conversant in this jargon.* Define or describe the following terms, use sketches if appropriate, and cite your sources (URL’s are acceptable if cited): struts, gusset, plenum, strain gage (*single element* and *strain gage rosette*), OEM (Original Equipment Manufacturer).