I have verified that this exam contains 9 problems and 6 printed pages. Initial___.

Print the name of the people sitting either side of you :- ________________

Short Answer (8 points each) - no explanation or calculations necessary though where appropriate, answers should be exact. Print your answers to each question in the appropriate numbered box below.

1. Find a formula for the \( n \)th term of the sequence

\[
\left\{ \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, \ldots \right\}.
\]

2. The series

\[
\sum_{n=1}^{\infty} ar^n
\]

where \( r \) and \( a \) are constants is called a geometric series.

(a) For what values of \( r \) does it converge?

(b) When it converges, what does it converge to (in terms of \( a \) and \( r \))?
3. The series
\[ \sum_{i=1}^{\infty} \frac{1}{n^p} \]
where \( p \) is a constant is called a \( p \)-series. For what values of \( p \) does it converge?

4. Briefly explain the difference between the limit comparison test and the ratio test for convergence of series.
5. **Briefly** explain why any power series centered at 0,
\[ \sum_{n=0}^{\infty} c_n x^n \]
always converges for at least one value of \( x \).

6. Write down an alternating series which does not converge.
Long Answer (18 points each) - show work and provide explanations, an answer without supporting work is not worth much.

1. Test the following series for convergence or divergence and state which test you are using:

(a) \[ \sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!} \]

(b) \[ \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n - 1} \]

(c) \[ \sum_{n=0}^{\infty} \frac{1 + \sin(n)}{10^n} \]
2. Find the radius of convergence and the interval of convergence of the series

\[ \sum_{n=1}^{\infty} (-1)^n n^4 x^n. \]
3. (a) For which values of $p$ does the integral
\[ \int_0^\infty e^{px} \, dx \]
converge? Find a formula in terms of $p$ for the values of the integral for which it converges.

(b) For which values of $p$ does the sequence
\[ a_n = e^{pn} \]
converge?

(c) For which values of $p$ does the series
\[ \sum_{k=1}^{\infty} e^{pk} \]
converge?