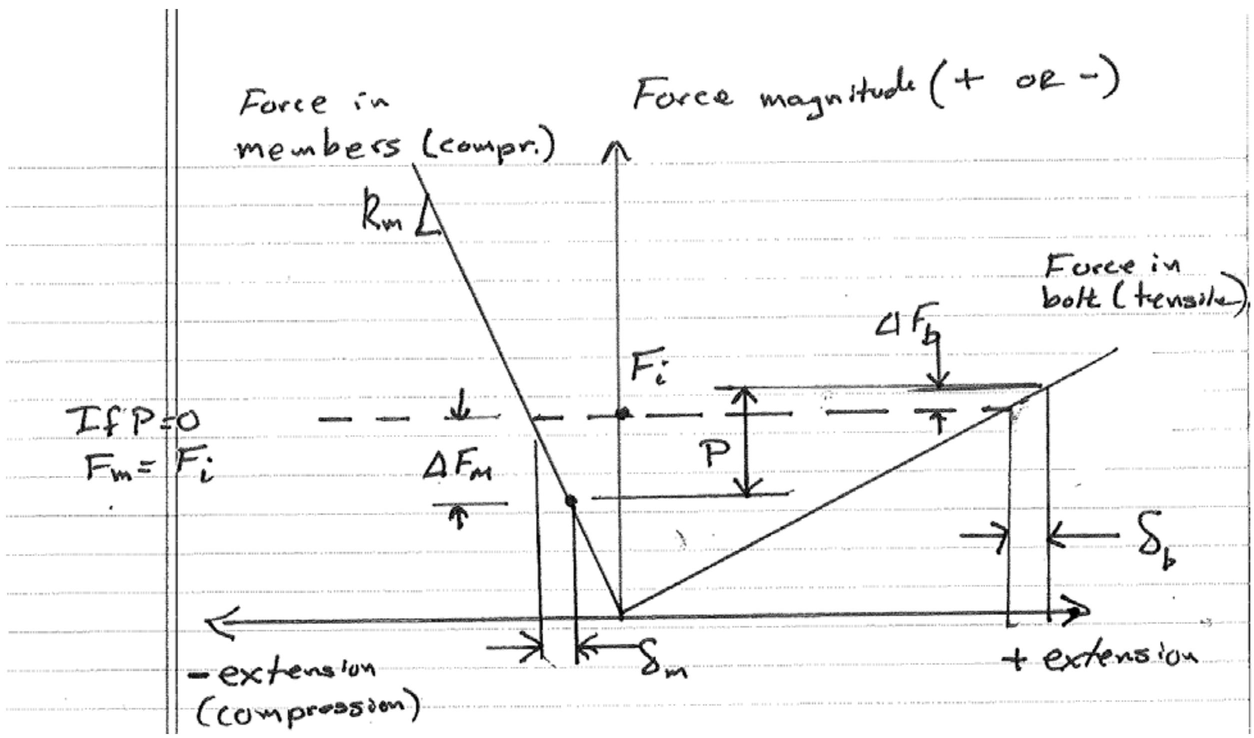
F_i = preload in the fastener
F_b = force in the bolt (due to preload and external load)
F_m = clamp load in the members. Typically expressed as "positive" even though it is a compressive load.
P = external applied load to the joint.
For P = 0; $F_m = F_b = F_i$
δ = change in length
k_b, k_m – stiffness of bolt and of members, respectively (remember, everything is a spring)
Let C be the so-called *joint stiffness constant*: $C = k_b / (k_b + k_m)$ - this is the proportion of the load carried by the fastener. Confused? Read on, this is critically important…

What if P < 0 (i.e. *compressive external load* on the joint)?
For P < 0; $k_m \sim \infty$ Why? Is there a fundamental difference between "opening a book" (tensile load) verses "pushing" the pages together (compressive load)? What if two of the pages were glued together – would the glue be useful in carrying any of the compressive load? Therefore; if P < 0, then $k_m \sim \infty$, which follows that $C \sim 0$ (no load is carried by the fastener). From that fact, the force in the members is equal to the preload plus the compressive applied load ($F_m \sim F_i + P$) and the force in the bolt is unaffected by the compressive load: $F_b \sim F_i$

The main focus of our study, is for P > 0 (*tensile external load*). For P > 0:
This graph is for tensile external loads (i.e. P > or = 0), and also only if there is ***no*** joint separation ($P < P_0$).



Using the graph:

δ_m = change in length of members due to P ($\delta_m > 0$)
δ_b = change in length of bolt due to P ($\delta_b > 0$)
$\delta_m = \delta_b = \delta$ if joint remains intact (does not separate)

The change in the force in the bolt when the external load is applied, ΔF_b is:
$\Delta F_b = k_b\delta$ and $\Delta F_m = k_m\delta$
$P = \Delta F_b + \Delta F_m$ (the external force, P, causes a change in force in the bolt and member).
$P = \Delta F_b + \Delta F_m = k_b\delta + k_m\delta = (k_b + k_m)\delta$ therefore, $\delta = P/(k_b + k_m)$

$\Delta F_b = \delta k_b = P\, k_b / (k_b + k_m)$ and $\Delta F_m = \delta k_m = P\, k_m / (k_b + k_m)$

Finally, we can determine the **force in the bolt and members when an external load (P) is applied**:

$\mathbf{F_b} = F_i + \Delta F_b = \mathbf{F_i + P\, k_b / (k_b + k_m)}$ and $\mathbf{F_m} = F_i - \Delta F_m = \mathbf{F_i - P\, k_m / (k_b + k_m)}$

NOTE: $F_b < F_i + P$!!! The joined members actually carry most of the external force (assuming no joint separation). Once the joint does separate (which they should be designed NOT to do!), then the bolt will carry all the external force: $F_b = P$.

What is k_b and k_m? k_m (member stiffness) is given in Shigley in eq 8-23 as:
$\mathbf{k_m = EdA\, exp(Bd/l)}$ { note: $\exp(Bd/L_{grip}) = e^{(Bd/L_{grip})}$ } where E is Young's modulus of the clamped members, d is the nominal fastener diameter, A and B are values given in Table 8-8, and L_{gr} is the grip length of the joint. k_b is also given in Shigley, but it is more complicated than it is justified. We will use the following. For an axially loaded member, the elongation, δ is:
$\delta = FL/EA$, so stiffness of this is $k = F/\delta = EA/L$. Putting this in fastener nomenclature:
$\mathbf{k_b = E_b A_b / L_{grip}}$ This equation is in bold here because it is NOT in the textbook – the book offers a bit more precise, but also more tedious method (it does provide a better answer, and the textbook method should be followed for actual design work). E_b is Young's modulus of the fastener, A_b is the cross section of the bolt $\{A_b = (\pi/4)d^2\}$ and L_{grip} is the grip length of the joint.

Let C be the so-called *joint stiffness constant*: $\mathbf{C = k_b / (k_b + k_m)}$
The joint constant (C) is the fraction of the load carried by the bolt (typically around 15 to 25%).

$\Delta F_b = PC$ and $\mathbf{F_b = F_i + PC}$
$\Delta F_m = P(1-C)$ and $\mathbf{F_m = F_i - P(1-C)}$ {**or *depending upon sign convention*: $\mathbf{F_m = P(1-C) - F_i}$** }

STATIC failure of tension joints typically involve joint separation (no clamp force between the members) and/or **permanent elongation of the fastener** ("yielding").

<u>Joint Separation:</u>

Let P_0 be the external force required for joint separation (not a good thing). When the joint separates, there is no longer any force carried through the members; therefore, $F_m = 0$ when P_0 is applied.

If $P < P_0$, then $F_m = F_i - P(1-C)$ now let $P = P_0$ and therefore, $F_m = 0$.

$F_m = 0 = F_i - P_0(1-C)$ This shows us that the force required to cause joint separation is:

$P_0 = F_i / (1-C)$

The factor of safety against joint separation, **n_0 or $n_{sep} = P_0 / P = F_i / P(1-C)$**

<u>Fastener Elongation ("yielding" = exceeding the proof strength)</u>

There are two common ways to define FOS (factor of safety) against fastener elongation (aka fastener yielding). Remember, FOS is required because we are ignorant. The more ignorant we are, the larger the FOS warranted.

The axial stress in the fastener's threaded region is $\sigma_b = F_b/A_t$ (A_t is the tensile stress area – see Tables 8.1 and 8.2). Axial stress is greater in the threaded region since the cross-sectional area is less there than in the solid shank of the fastener.

$\sigma_b = F_b/A_t = (F_i + PC)/A_t$

FOS for exceeding the proof strength: **$n_p = S_p / \sigma_b = S_pA_t/(F_i + PC)$ (eq 8-28)**

However, we often have fair confidence in knowing the preload (Fi), it is the applied load (external force) that we often have greater ignorance about. **If we apply the FOS (ignorance factor) only to P, we get what is called the Load Factor (n_L). Applying the FOS to the external load only (and not the preload):**

$S_p = (n_LCP + F_i)/A_t$ And solving for the FOS (n_L): **$n_L = (S_pA_t - F_i) / CP$ (eq 8-29)**

In other words, the external load can be n_LP (n_L times greater than the expected load, P) without causing permanent elongation of the fastener.