the last equality holding by virtue of $(\partial y/\partial t)dt = -(\partial y/\partial x)dx$ for the traveling wave, which permits us to convert the time integral to an integral over x at time t (whereas the time integrals were carried out at $x=x_1$). This integral, inserted into Eq. (3), just cancels half the first integral on the right-hand side of the equation, leaving the result

$$G_x = -\frac{1}{2}\lambda_0 \int_{x_1}^{x_2} \frac{\partial \eta}{\partial t}\frac{\partial \eta}{\partial x}dx, \quad (6)$$

which leads to identification of

$$g_x = -\frac{1}{2}\lambda_0 \frac{\partial \eta}{\partial t}\frac{\partial \eta}{\partial x} \quad (7)$$

as the momentum density. This is the result obtained by Rowland and Pask, under the usual conditions pertaining to transverse waves on a string.

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[2]W. C. Elmore and M. A. Heald, *Physics of Waves* (McGraw–Hill, New York, 1969; Dover, New York, 1985).

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Impedance between adjacent nodes of infinite uniform *D*-dimensional resistive lattices

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Infinite resistive network problems have served as excellent vehicles for helping electrical engineering and physics students recognize and appreciate the power of superposition and symmetry in the analysis of electrical networks. These problems have been studied extensively using superposition and symmetry.[1–10] A special case of this class of problems involves the calculation of the effective resistance between two adjacent nodes of an infinite uniform two-dimensional (2D) resistive lattice (periodic in both dimensions with a zero-potential boundary condition at infinity) comprised of identical resistors each of value R. In particular, the effective resistance between two adjacent nodes of the 2D Liebman resistive mesh (the infinite 2D square resistive lattice) was calculated by Aitchison[1] and found to be $(1/2)R$. Bartis[2] calculated the resistance between adjacent nodes for three other infinite 2D resistive lattices, the triangular, Honeycomb, and Kagomé lattices, and found the effective resistances to be $(1/3)R$, $(2/3)R$, and $(1/2)R$, respectively.

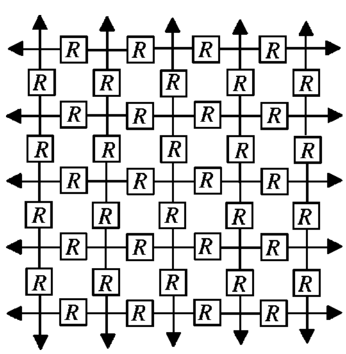

Fig. 1. Infinite 2D square resistive lattice.

The goal of this paper is to extend the results of Refs. 1 and 2 to the general problem of finding the total effective resistance R_{eff} between two adjacent nodes of any infinite D-dimensional resistive lattice, where $D=1, 2, 3,...$ and the lattice is periodic and infinite in all D dimensions with a zero-potential boundary condition at infinity. Our general solution for R_{eff} is of pedagogical interest because it generalizes the previous results of Refs. 1 and 2 to a simple and elegant equation that covers all adjacent-node infinite D-dimensional resistive networks and because it reinforces the power of the superposition principle and symmetry in electrical circuit analysis.

For the purpose of illustration, consider the infinite 2D square resistive lattice shown in Fig. 1. The number of resistors connected to each node is denoted by M ($M=4$ in Fig. 1). As in Refs. 1 and 2, we use superposition and symmetry along with two test current sources each of value I to calculate the effective resistance R_{eff} between two adjacent nodes by injecting a test current I into any single node on the D-dimensional resistive lattice from the zero-potential boundary at infinity and then extracting another identical test current I from an adjacent node connected to a current sink kept at zero potential. By using Kirchhoff's current law and symmetry, we find that each of the M resistors connected to the original node will receive I/M of the injected current. Similarly, we find that each of the M resistors connected to the adjacent node will receive $-I/M$ of the extracted current in the opposite direction. Therefore, by superposition, the total resulting current flowing in the resistor R connecting the two adjacent nodes will be $2I/M$, which leads to a voltage drop across the resistor R of $V=(2I/M)R$. Thus the effective resistance is

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$$R_{\text{eff}} = V/I = 2R/M. \quad (1)$$

Equation (1) is a new and remarkably simple, elegant, and powerful result that applies to any infinite D-dimensional resistive lattice.

As an aside, we note that using symmetry, superposition, and a Laplacian analysis, the corresponding effective impedances for an infinite D-dimensional purely inductive or capacitive lattice can be determined in a similar way as $L_{\text{eff}} = 2L/M$ and $C_{\text{eff}} = MC/2$, respectively. These results are new and equally simple, elegant, and interesting (especially C_{eff} due to its different and nonintuitive dependence on M).

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