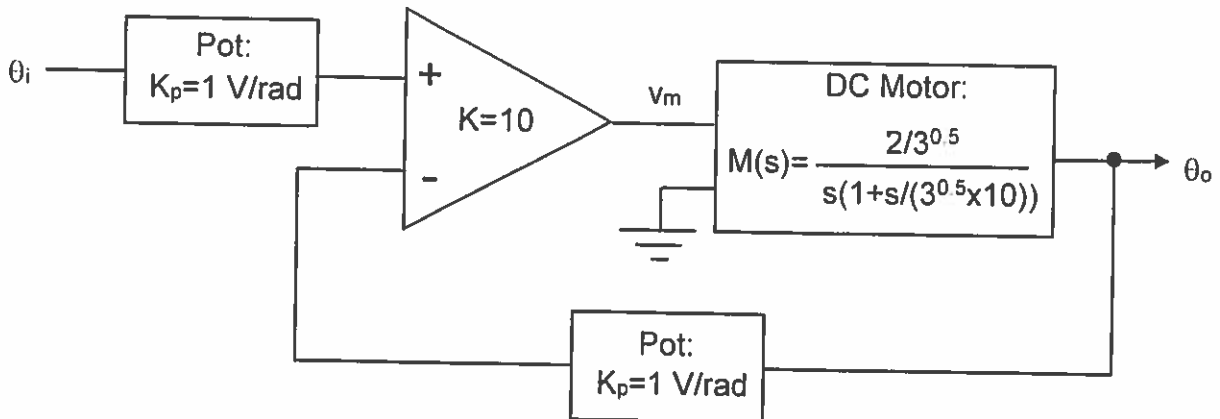


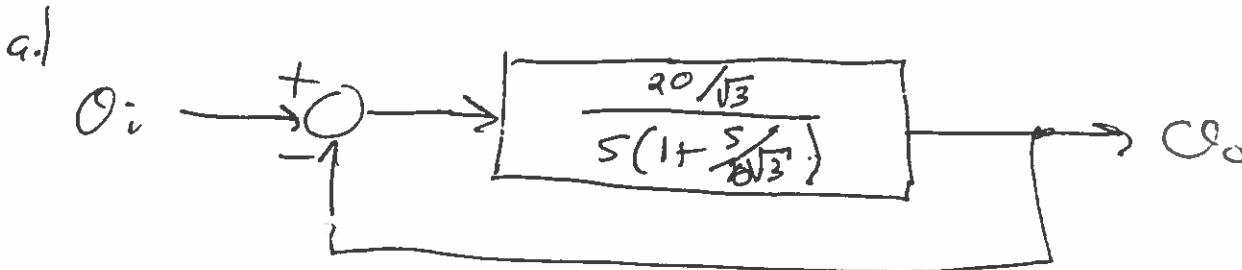
Problem 1 (50 points):

Consider the following closed-loop negative feedback servo position control system comprised of an ideal opamp with a constant gain of $K=10$, a DC motor with the given transfer function, $M(s)=\theta_o/V_m(s)$, and two potentiometers both with $K_p=1\text{V/rad}$.



- Sketch the equivalent feedback block diagram of this closed-loop servo system.
- Determine the closed-loop transfer function, $A_f(s)=\theta_o/\theta_i(s)$. (Be sure to properly "simplify" your final expression).
- Determine the Loop Gain, $LG(s)$.
- Calculate the PM.
- Optional extra credit (10 points, no partial credit):**

Calculate the GM. (Please justify your answer by showing your work).



b.)
$$A_f(s) = \frac{20\sqrt{3}/s(1+s/10\sqrt{3})}{1 + \frac{20\sqrt{3}}{s(1+s/10\sqrt{3})}}$$

~~$$A_f(s) = \frac{20\sqrt{3}}{s^2 + \frac{s}{20\sqrt{3}} + 1}$$~~

$$A_f(s) = \frac{1}{\frac{s^2}{200} + \frac{\sqrt{3}}{20}s + 1}$$

$$c.) \quad LG(s) = \frac{\frac{20}{\sqrt{3}}}{s(1 + \frac{s}{10\sqrt{3}})}$$

$$d.) \quad PM = 180^\circ - |\angle LG(j\omega_1)|$$

$$|LG(j\omega_1)| = 1 = \frac{\left(\frac{20}{\sqrt{3}}\right)^2}{\omega_1^2 \left[1 + \frac{\omega_1^2}{300}\right]}$$

$$\frac{\omega_1^4}{300} + \omega_1^2 - \left(\frac{20}{\sqrt{3}}\right)^2 = 0$$

$$\omega_1^2 = \frac{-1 \pm \sqrt{1 + \frac{4(400)}{3(300)}}}{\frac{2}{300}} = \frac{-1 \pm \frac{5}{3}}{\frac{2}{300}}$$

$$\omega_1^2 = \frac{2/3}{2/300} = 100 \Rightarrow \omega_1 = 10 \text{ rad/sec}$$

$$\angle LG(j\omega_1) = -90^\circ - \tan^{-1}\left(\frac{10}{\sqrt{3}}\right)$$

$$= -90^\circ - 30^\circ = -120^\circ$$

$$\therefore PM = 180^\circ - |-120^\circ| = 60^\circ$$

$$e.) \quad GM = 20 \log \left[\frac{1}{|LG(j\omega_{180})|} \right] = \infty$$

~~LG~~ LG never gets to 180°

$$\omega_{180} = \infty \Rightarrow |LG(j\omega_{180})| = 0$$

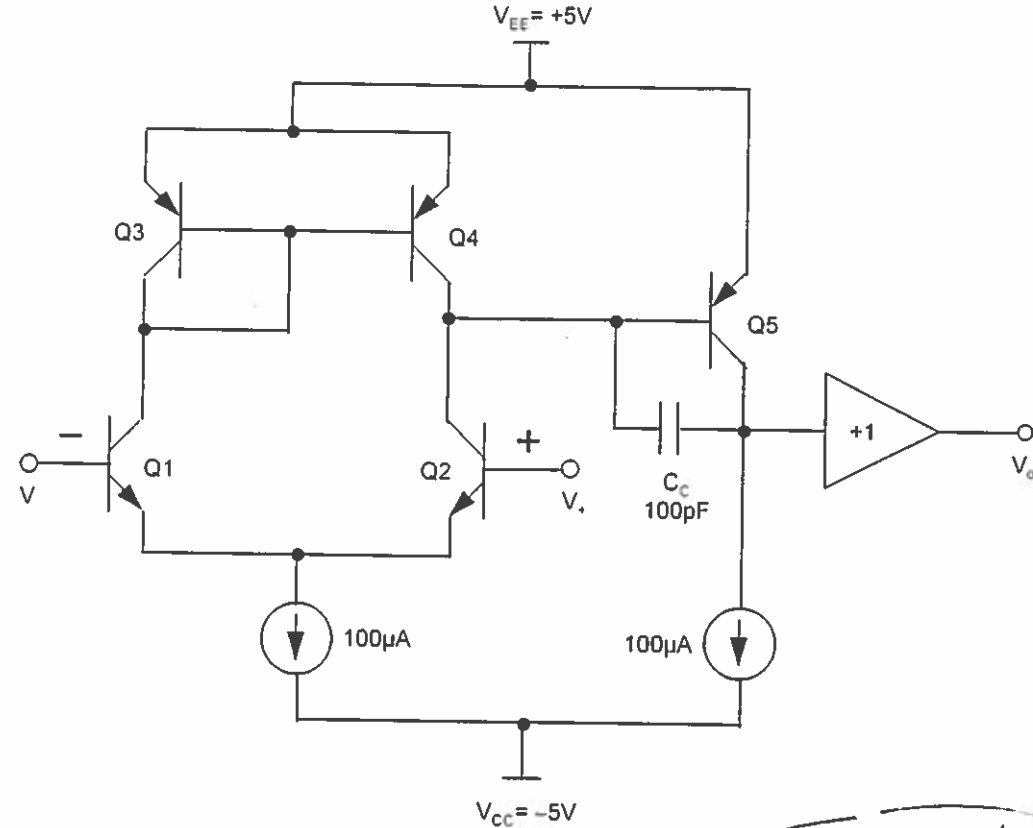
$$|\angle LG(j\omega_{180})| = 180 = \left| -90^\circ - \tan^{-1} \frac{\omega_{180}}{10\sqrt{3}} \right|$$

$$\Rightarrow 90^\circ = \tan^{-1} \frac{\omega_{180}}{10\sqrt{3}}$$

$$\Rightarrow \omega_{180} = \infty$$

Problem 2 (50 points)

Consider the following open-loop opamp circuit. Assume the unity-gain output buffer is ideal. Assume $V_T=25\text{mV}$, $\beta_n=\beta_p=100$, and $V_{A_n}=V_{A_p}=100\text{V}$. You may make reasonable approximations in your calculations.



a) Calculate ω_t of the opamp.

$$\omega_t = \frac{g_{m2}}{C_c} = 20\text{ Mrad/sec}$$

b) Calculate SR of the opamp.

$$SR = I_{o1} / C_c = 1\text{ V}/\mu\text{s}$$

c) Now connect the opamp in its closed-loop, unity-gain configuration. Determine the closed-loop transfer function, $A_f(s)$. (You may assume that $A(s) = \omega_t/s$).

$$A_f(s) = \frac{1}{1 + s/\omega_t} = \frac{1}{1 + s/20 \times 10^6}$$

d) With the opamp still in its closed-loop, unity-gain configuration, if $v_i(t)$ is a square wave of amplitude $\pm 1\text{V}$ and frequency 250kHz , sketch and label the output voltage, $v_o(t)$.

Next page

e) **Optional extra credit (10 points, no partial credit):**

Calculate A_0 and ω_H of the opamp.

$$A_0 = g_{m2} [r_{o2} \parallel r_{o4} \parallel r_{\pi5}] g_{m5} r_{o5} = 200\text{ kV/V}$$

$r_{\pi5} = 25\text{ k}$

$$\omega_H = \frac{\omega_t}{A_0} = 100\text{ rad/sec}$$

