The University of Portland
Donald P. Shiley School of Engineering

EE451
Advanced Analog Electronics

HOMEWORK 1

Assigned: Tues, Aug 25, 2020
Due: Tues, Sept 8, 2020

Problems:

Note: It may be helpful to consult S&S Table 3.1 and J&M Sec 1.5 for some of these problems.

1) S&S Text 3.6 (parts a, b, c, and e only). Note, parts a and b should read as follows:

   a) intrinsic silicon
   b) n-doped silicon with $N_D=10^{16}/cm^3$

2) S&S Text 3.22

3) An npn transistor has an emitter area of 10um x 10um. The doping concentrations are as follows: in the emitter $N_D=10^{19}/cm^3$, in the base $N_A=10^{17}/cm^3$, and in the collector $N_D=10^{15}/cm^3$. The transistor is operating at T=300K, where $n_i=1.5 \times 10^{10}/cm^3$. For electrons diffusing in the base, L_n=19um and $D_n=21.3cm^2/s$. For holes diffusing in the emitter, L_p=0.6um and $D_p=1.7cm^2/s$. Calculate I_S and β assuming that the base-width W is:

   a) 1um
   b) 2um
   c) 5um

   For case b), if I_C=1mA, find I_B, I_E, and V_{BE}.

4) J&M Text 1.9 and 1.10 (Note that for these problems you will need the given parameters on pp. 78-79. They're easy to miss. Also, always assume that $g_s=0$ in the MOSFET SS hybrid-pi model. Finally, assume that the MOSFET is an n-channel type in Problem 1.10).

5) Simulate the NMOS circuit on the following page in PSPICE (using Transient Response analysis) to obtain the large-signal step response, $V_o(t)$. Assume the following NMOS parameters and conditions: W=100μm, L=0.5μm, $KP_n=\mu_n C_{ox}=100\mu A/V^2$, $V_{tn}=0.9V$, $\gamma=0V^{0.5}$, $\lambda=0V^{-1}$, COX=1.9E-3pF/μm², CJ=2.4E-4pF/μm², CJSW=CGSO=CGDO=2E-4pF/μm, and $V_i(t)$ steps from 0 to 1V instantaneously. Remember that you get TOX from COX. Assume all other parameters default. You should use an MbreakN3 and edit its model accordingly. Don't forget AD, AS, PD, and PS. You should also use a VPULSE source. Please hand-in your PSPICE printouts including your Schematic and PROBE output showing $V_o(t)$ and $V_i(t)$. Please hand-comment your PSPICE printouts appropriately

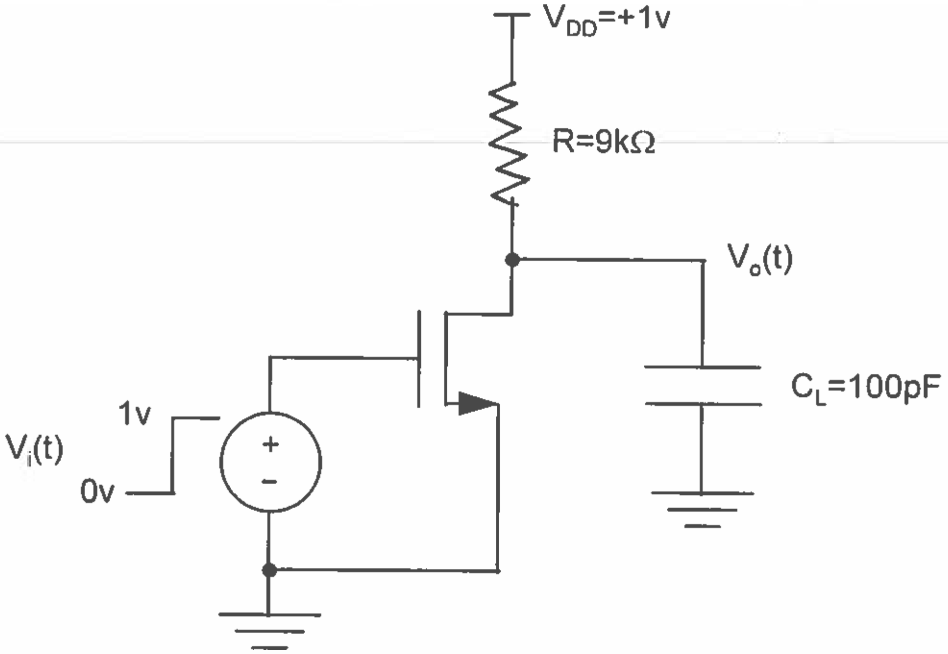
V_{DD}=+1v
R=9kΩ
$V_o(t)$
C_L=100pF
1v
$V_i(t)$
0v
+
-

Table 3.1 Summary of Important Equations

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	$B = 7.3 \times 10^{15}\text{cm}^{-3}\text{K}^{-3/2}$ $E_g = 1.12$ eV $k = 8.62 \times 10^{-5}$eV/K $n_i = 1.5 \times 10^{10}/\text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p\frac{dp}{dx}$ $J_n = qD_n\frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{drift} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega\cdot$cm)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \simeq 25.8$ mV
Carrier concentration in *n*-type silicon (cm^{-3})	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in *p*-type silicon (cm^{-3})	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\varepsilon_s = 11.7\varepsilon_0$ $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm
Charge stored in depletion layer (coulomb)	$Q_J = q\frac{N_A N_D}{N_A + N_D}AW$	
Forward current (A)	$I = I_p + I_n$ $I_p = Aqn_i^2\frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$ $I_n = Aqn_i^2\frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$	
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
I-V Relationship	$I = I_S(e^{V/V_T} - 1)$	

Table 3.1 *continued*

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \qquad \tau_n = L_n^2/D_n$	$L_p, L_n = 1\ \mu\text{m to } 100\ \mu\text{m}$ $\tau_p, \tau_n = 1\text{ ns to } 10^4\text{ ns}$
Minority-carrier charge storage (coulomb)	$Q_p = \tau_p I_p \qquad Q_n = \tau_n I_n$ $Q = Q_p + Q_n = \tau_T I$	
Depletion capacitance (F)	$C_{j0} = A\sqrt{\left(\frac{\varepsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$ $C_j = C_{j0}\Big/\left(1 + \frac{V_R}{V_0}\right)^m$	 $m = \frac{1}{3}\text{ to }\frac{1}{2}$
Diffusion capacitance (F)	$C_d = \left(\frac{\tau_T}{V_T}\right)I$	

PROBLEMS

Problems are marked with asterisks to describe their degree of difficulty. Difficult problems are marked with an asterisk (*); more difficult problems with two asterisks (**); and very challenging and/or time-consuming problems with three asterisks (***). Also, if in the following problems the need arises for the values of particular parameters or physical constants that are not stated, please consult Table 3.1.

Section 3.1: Intrinsic Semiconductors

3.1 Find values of the intrinsic carrier concentration n_i for silicon at −70°C, 0°C, 20°C, 100°C, and 125°C. At each temperature, what fraction of the atoms is ionized? Recall that a silicon crystal has approximately 5×10^{22} atoms/cm^3.

3.2 Calculate the value of n_i for gallium arsenide (GaAs) at $T = 300$ K. The constant $B = 3.56 \times 10^{14}$ (cm^{-3}K$^{-3/2}$) and the bandgap voltage $E_g = 1.42$ eV.

Section 3.2: Doped Semiconductors

3.3 For a p-type silicon in which the dopant concentration $N_A = 10^{18}/\text{cm}^3$, find the hole and electron concentrations at $T = 300$ K.

3.4 For a silicon crystal doped with phosphorus, what must N_D be if at $T = 300$ K the hole concentration drops below the intrinsic level by a factor of 10^7?

3.5 In a phosphorus-doped silicon layer with impurity concentration of $10^{16}/\text{cm}^3$, find the hole and electron concentrations at 27°C and 125°C.

Section 3.3: Current Flow in Semiconductors

3.6 A young designer, aiming to develop intuition concerning conducting paths within an integrated circuit, examines the end-to-end resistance of a connecting bar 10 μm long, 3 μm wide, and 1 μm thick, made of various materials. The designer considers:

~~(a) intrinsic silicon~~
~~(b) n-doped $N_D = 10^{16}/cm^3$~~
~~(c) n-doped $N_D = 10^{18}/cm^3$~~

(c) n-doped silicon with $N_D = 10^{18}/\text{cm}^3$

(d) p-doped silicon with $N_A = 10^{16}/\text{cm}^3$

(e) aluminum with resistivity of 2.8 $\mu\Omega \cdot \text{cm}$

Find the resistance in each case. For intrinsic silicon, use the data in Table 3.1. For doped silicon, assume $\mu_n = 2.5\mu_p = 1200\ \text{cm}^2/\text{V·s}$. (Recall that $R = \rho L/A$)

3.7 Contrast the electron and hole drift velocities through a 10-μm layer of intrinsic silicon across which a voltage of 5 V is imposed. Let $\mu_n = 1350\ \text{cm}^2/\text{V·s}$ and $\mu_p = 480\ \text{cm}^2/\text{V·s}$.

3.8 Find the current that flows in a silicon bar of 10-μm length having a 5-μm × 4-μm cross section and having free electron and hole densities of $10^5/\text{cm}^3$ and $10^{15}/\text{cm}^3$, respectively, when a 1 V is applied end-to-end. Use $\mu_n = 1200\ \text{cm}^2/\text{V·s}$ and $\mu_p = 500\ \text{cm}^2/\text{V·s}$.

3.9 In a 10-μm long bar of donor-doped silicon, what donor concentration is needed to realize a current density of 1 mA/μm^2 in response to an applied voltage of 1 V. (*Note*: Although the carrier mobilities change with doping concentration, as a first approximation you may assume μ_n to be constant and use the value for intrinsic silicon, 1350 cm^2/V·s).

3.10 Holes are being steadily injected into a region of n-type silicon (connected to other devices, the details of which are not important for this question). In the steady state, the excess-hole concentration profile shown in Fig. P3.10 is established in the n-type silicon region. Here "excess" means over and above the thermal-equilibrium concentration (in the absence of hole injection), denoted p_{n0}. If $N_D = 10^{16}/\text{cm}^3$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $D_n = 12\ \text{cm}^2/\text{s}$, and $W = 0.1$ μm, find the density of the current that will flow in the x direction.

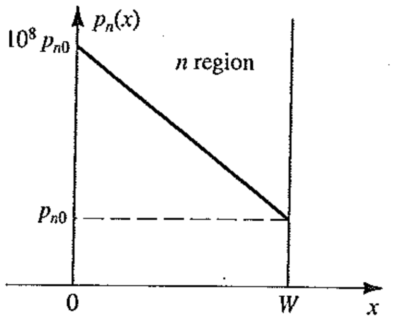

Figure P3.10

3.11 Both the carrier mobility and diffusivity decrease as the doping concentration of silicon is increased. The table below provides a few data points for μ_n and μ_p versus doping concentration. Use the Einstein relationship to obtain the corresponding values for D_n and D_p.

Section 3.4: The *pn* Junction with Open-Circuit Terminals (Equilibrium)

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3.12 Calculate the built-in voltage of a junction in which the p and n regions are doped equally with 10^{16} atoms/cm^3. Assume $n_i = 1.5 \times 10^{10}/\text{cm}^3$. With the terminals left open, what is the width of the depletion region, and how far does it extend into the p and n regions? If the cross-sectional area of the junction is 100 μm^2, find the magnitude of the charge stored on either side of the junction.

3.13 If, for a particular junction, the acceptor concentration is $10^{16}/\text{cm}^3$ and the donor concentration is $10^{15}/\text{cm}^3$, find the junction built-in voltage. Assume $n_i = 1.5 \times 10^{10}/\text{cm}^3$. Also, find the width of the depletion region (W) and its extent in each of the p and n regions when the junction terminals are left open. Calculate the magnitude of the charge stored on either side of the junction. Assume that the junction area is 400 μm^2.

Doping Concentration (carriers/cm^3)	μ_n (cm^2/V·s)	μ_p (cm^2/V·s)	D_n (cm^2/s)	D_p (cm^2/s)
Intrinsic	1350	480		
10^{16}	1100	400		
10^{17}	700	260		
10^{18}	360	150		

Table P3.11

3.14 Estimate the total charge stored in a 0.1-μm depletion layer on one side of a 10-μm × 10-μm junction. The doping concentration on that side of the junction is $10^{16}/\text{cm}^3$.

3.15 In a *pn* junction for which $N_A \gg N_D$, and the depletion layer exists mostly on the shallowly doped side with W = 0.3 μm, find V_0 if $N_D = 10^{16}/\text{cm}^3$. Also calculate Q_J.

3.16 By how much does V_0 change if N_A or N_D is increased by a factor of 10?

Section 3.5: The *pn* Junction with an Applied Voltage

3.17 If a 5-V reverse-bias voltage is applied across the junction specified in Problem 3.13, find W and Q_J.

3.18 Show that for a *pn* junction reverse-biased with a voltage V_R, the depletion-layer width W and the charge stored on either side of the junction, Q_J, can be expressed as

$$W = W_0\sqrt{1 + \frac{V_R}{V_0}}$$

$$Q_J = Q_{J0}\sqrt{1 + \frac{V_R}{V_0}}$$

where W_0 and Q_{J0} are the values in equilibrium.

3.19 In a forward-biased *pn* junction show that the ratio of the current component due to hole injection across the junction to the component due to electron injection is given by

$$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D}$$

Evaluate this ratio for the case $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $L_p = 5$ μm, $L_n = 10$ μm, $D_p = 10\ \text{cm}^2/\text{s}$, and $D_n = 20\ \text{cm}^2/\text{s}$, and hence find I_p and I_n for the case in which the *pn* junction is conducting a forward current I = 1 mA.

3.20 Calculate I_S and the current I for V = 700 mV for a *pn* junction for which $N_A = 10^{17}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, A = 200 μm², $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $L_p = 5$ μm, $L_n = 10$ μm, $D_p = 10\ \text{cm}^2/\text{s}$, and $D_n = 18\ \text{cm}^2/\text{s}$.

3.21 Assuming that the temperature dependence of I_S arises mostly because I_S is proportional to n_i^2, use the expression for n_i in Eq. (3.2) to determine the factor by which n_i^2 changes as T changes from 300 K to 305 K. This will be approximately the same factor by which I_S changes for a 5°C rise in temperature. What is the factor?

3.22 A p^+n junction is one in which the doping concentration in the *p* region is much greater than that in the *n* region. In such a junction, the forward current is mostly due to hole injection across the junction. Show that

$$I \simeq I_p = Aqn_i^2\frac{D_p}{L_pN_D}(e^{V/V_T} - 1)$$

For the specific case in which $N_D = 10^{16}/\text{cm}^3$, $D_p = 10\ \text{cm}^2/\text{s}$, $L_p = 10$ μm, and $A = 10^4$ μm², find I_S and the voltage V obtained when I = 0.5 mA. Assume operation at 300 K where $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

3.23 A *pn* junction for which the breakdown voltage is 12 V has a rated (i.e., maximum allowable) power dissipation of 0.25 W. What continuous current in the breakdown region will raise the dissipation to half the rated value? If breakdown occurs for only 10 ms in every 20 ms, what average breakdown current is allowed?

Section 3.6: Capacitive Effects in the *pn* Junction

3.24 For the *pn* junction specified in Problem 3.13, find C_{j0} and C_j at $V_R = 5$ V.

3.25 For a particular junction for which $C_{j0} = 0.6$ pF, $V_0 = 0.75$ V, and m = 1/3, find C_j at reverse-bias voltages of 1 V and 10 V.

3.26 The junction capacitance C_j can be thought of as that of a parallel-plate capacitor and thus given by

$$C_j = \frac{\varepsilon A}{W}$$

Show that this approach leads to a formula identical to that obtained by combining Eqs. (3.43) and (3.45) [or equivalently, by combining Eqs. (3.47) and (3.48)].

3.27 A *pn* junction operating in the forward-bias region with a current I of 1 mA is found to have a diffusion capacitance of 10 pF. What diffusion capacitance do you expect this junction to have at I = 0.1 mA? What is the mean transit time for this junction?

3.28 For the p^+n junction specified in Problem 3.22, find τ_p and calculate the excess minority carrier charge and the value of the diffusion capacitance at I = 0.2 mA.